

Simple scheme for two-qubit Grover search in cavity QED

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Abstract

Following the proposal by F. Yamaguchi et al.[Phys. Rev. A **66**, 010302 (R) (2002)], we present an alternative way to implement the two-qubit Grover search algorithm in cavity QED. Compared with F. Yamaguchi et al.'s proposal, with a strong resonant classical field added, our method is insensitive to both the cavity decay and thermal field, and doesn't require that the cavity remain in the vacuum state throughout the procedure. Moreover, the qubit definitions are the same for both atoms, which makes the experiment easier. The strictly numerical simulation shows that our proposal is good enough to demonstrate a two-qubit Grover's search with high fidelity.

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The Grover search algorithm[1] is an efficient quantum algorithm to look for one item in an unsorted database of size N . While the most efficient classical algorithm which examines items one by one needs on average $N/2$ queries, the Grover's quantum algorithm uses only $O(\sqrt{N})$ queries to accomplish the same task. The efficiency of this algorithm has been tested experimentally in few-qubit cases by NMR [2] and by optics [3].

Grover's search can be briefly described as follows: For items represented by the computational states $|X\rangle$ with $X = 0, 1, \dots, N-1$, in a quantum register with n qubits, we have $N = 2^n$ possible states. The search starts from a superposition state $|\Psi_0\rangle = \frac{1}{\sqrt{N}} \sum_{X=0}^{N-1} |X\rangle$, where each item has an equal probability to get picked. One search step (i.e., a query) includes two key operations [4]: (i) Inverting the amplitude of the target item; (ii) Performing a diffusion transform D , i.e., inversion about the average state $|\Psi_0\rangle$ with $D_{ij} = 2/N$ for $i \neq j$ and $D_{ii} = -1 + 2/N$. If the target item is $|\tau\rangle$, then the operation in case (i) results in a conditional phase gate $I_\tau = I - 2|\tau\rangle\langle\tau|$, where I is the $N \times N$ identity matrix. After $O(\sqrt{N})$ queries, the amplitude of the identified target would be amplified while amplitude of non-target items are shrunk to be negligible. Thus we get the target item with high probability.

Various schemes have been proposed for implementing several quantum algorithms in cavity QED, for example, Grover search algorithm[5], quantum discrete Fourier transform[6], Deutsch-Jozsa algorithm[7], quantum dense coding[8] and so on. Although cavity QED is one of the qualified candidates for quantum information processing(QIP) and attracts much attention, decoherence of the cavity field remains to be a big obstacle for QIP in cavity QED. Recently, Zheng and Guo proposed two atoms interacting with a nonresonant cavity, in which the two atoms can be entangled without information transfer between atoms and the cavity[9]. However, it requires the cavity to be initially in the vacuum state. Osnaghi et al.[10] have experimentally demonstrated this scheme. Yamaguchi et al. extended Zheng and Guo's proposal to a scheme to realize the two-qubit Grover search algorithm in cavity QED. In order to achieve the quantum phase gate, they have to choose different levels for qubit encoding for the two atoms[5]. In Refs. [8, 11], with a strong resonant classical field added, the photon-number-dependent Stark shift can be canceled. It does not require that the cavity be initially in the vacuum state, and the scheme is insensitive to both the cavity decay and the thermal field.

In this paper, we modify the proposal in Ref. [5] by adding a strong resonant classical

field. Comparing with Ref. [5], we find our method have following merits: (i) Initial vacuum cavity field is not needed, and our method is insensitive to the thermal field besides cavity decay; (ii) Qubit definitions are the same for the two atoms, which makes the experiment easier; (iii) The two-qubit gate can be easily achieved by an appropriate Rabi frequency Ω (defined below); (iiii) Except for two NOT gates on atom 2 for labeling target state $|g_1\rangle|e_2\rangle$ or $|e_1\rangle|g_2\rangle$, all the other operations are simultaneously imposed on the two atoms, which makes the implementation more compact.

We consider two identical two-level atoms simultaneously interacting with a single-mode cavity field and driven by a classical field. The Hamiltonian (assuming $\hbar = 1$) in the rotating-wave approximation reads[8, 11]

$$H = \frac{1}{2} \sum_{j=1}^2 \omega_0 \sigma_{z,j} + \omega_a a^\dagger a + \sum_{j=1}^2 [g(a^\dagger \sigma_j^- + a \sigma_j^+) + \Omega(\sigma_j^+ e^{-i\omega t} + \sigma_j^- e^{i\omega t})] \quad (1)$$

where $\sigma_{z,j} = |e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|$, $\sigma_j^+ = |e_j\rangle\langle g_j|$, $\sigma_j^- = |g_j\rangle\langle e_j|$, with $|e_j\rangle$ ($|g_j\rangle$) being the excited (ground) state of the j th atom. ω_0 , ω_a , ω are the frequency for atomic transition, cavity mode and classical field respectively. a^\dagger , a are the creation and annihilation operators for the cavity mode. g is the atom-cavity coupling strength and Ω is the Rabi frequency of the classical field. Assuming $\omega_0 = \omega$ and $\delta = \omega_0 - \omega_a$, we have following Hamiltonian in the interaction picture [8, 11]

$$H_I = \sum_{j=1}^2 [\Omega(\sigma_j^+ + \sigma_j^-) + g(e^{-i\delta t} a^\dagger \sigma_j^- + e^{i\delta t} a \sigma_j^+)] \quad (2)$$

When $\Omega \gg \delta$, g and $\delta \gg g$, we can get the evolution operator of the system in the interaction picture[8, 11]

$$U_I(t) = e^{-iH_0 t} e^{-iH_e t} \quad (3)$$

with

$$H_0 = \sum_{j=1}^2 \Omega(\sigma_j^+ + \sigma_j^-) \quad (4)$$

$$H_e = \lambda \left[\frac{1}{2} \sum_{j=1}^2 (|e_j\rangle\langle e_j| + |g_j\rangle\langle g_j|) + (\sigma_1^+ \sigma_2^+ + \sigma_1^+ \sigma_2^- + H.c.) \right] \quad (5)$$

where $\lambda = g^2/2\delta$. It's obvious from Eq. (5), the phonon-number dependent Stark shift has been canceled by an additional strong resonant classical field. If we define $J_x = \frac{1}{2} \sum_{j=1}^2 (\sigma_j^+ + \sigma_j^-)$, Eqs. (4) and (5) reduce to $H_0 = 2\Omega J_x$, $H_e = 2\lambda J_x^2$ respectively and Eq. (3) becomes

$$\begin{aligned} U_I(t) &= e^{-i2\Omega t J_x} e^{-i2\lambda t J_x^2} \\ &= e^{-ib(J_x^2 + hJ_x)} \end{aligned} \quad (6)$$

with $b = 2\lambda t$ and $h = \frac{\Omega}{\lambda}$. In the subspace spanned by $|e_1\rangle|e_2\rangle$, $|e_1\rangle|g_2\rangle$, $|g_1\rangle|e_2\rangle$, $|g_1\rangle|g_2\rangle$, we define the two-qubit Hadamard gate

$$\begin{aligned} H^{\otimes 2} &= \prod_{i=1}^2 H_i \\ &= \left(\frac{1}{\sqrt{2}}\right)^2 \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{aligned} \quad (7)$$

where H_i is the Hadamard gate acting on the i th atom, transforming states as $|g_i\rangle \rightarrow \frac{1}{\sqrt{2}}(|g_i\rangle + |e_i\rangle)$, $|e_i\rangle \rightarrow \frac{1}{\sqrt{2}}(|g_i\rangle - |e_i\rangle)$. The $U_I(t)$ can be expressed in the same basis as:

$$U_I(t) = \begin{bmatrix} \frac{1}{2} + \frac{1}{2} \cos(bh)e^{-ib} & \frac{-i}{2} \sin(bh)e^{-ib} & \frac{-i}{2} \sin(bh)e^{-ib} & \frac{-1}{2} + \frac{1}{2} \cos(bh)e^{-ib} \\ \frac{-i}{2} \sin(bh)e^{-ib} & \frac{1}{2} + \frac{1}{2} \cos(bh)e^{-ib} & \frac{-1}{2} + \frac{1}{2} \cos(bh)e^{-ib} & \frac{-i}{2} \sin(bh)e^{-ib} \\ \frac{-i}{2} \sin(bh)e^{-ib} & \frac{-1}{2} + \frac{1}{2} \cos(bh)e^{-ib} & \frac{1}{2} + \frac{1}{2} \cos(bh)e^{-ib} & \frac{-i}{2} \sin(bh)e^{-ib} \\ \frac{-1}{2} + \frac{1}{2} \cos(bh)e^{-ib} & \frac{-i}{2} \sin(bh)e^{-ib} & \frac{-i}{2} \sin(bh)e^{-ib} & \frac{1}{2} + \frac{1}{2} \cos(bh)e^{-ib} \end{bmatrix} \quad (8)$$

If we choose $b = \frac{\pi}{2}$, $bh = \frac{\pi}{2} + 2m\pi$ (m is an interger), i.e., $\lambda t = \frac{\pi}{4}$, $\frac{\Omega}{\lambda} = 4m + 1$, we can get

$$U_I(t_D) = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{1}{2} \end{bmatrix} = -D \quad (9)$$

where $t_D = \frac{\pi}{4\lambda}$ [12]. So by choosing an appropriate value of Ω , we can generate a two-qubit diffusion transform D (different by -1 prefactor). The two-qubit conditional phase gate to label different target states will also be generated in a natural way. It's easy to have

$$H^{\otimes 2}U_I(t)H^{\otimes 2} = \begin{bmatrix} e^{ib(h-1)} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{-ib(h+1)} \end{bmatrix} \quad (10)$$

If we choose $b = \frac{\pi}{2}$, $h = 4m + 1$ (m is an interger), i.e., $t_1 = \frac{\pi}{4\lambda}$, $\frac{\Omega}{\lambda} = 4m + 1$, we have

$$H^{\otimes 2}U_I(t_1)H^{\otimes 2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = I_{g_1g_2} \quad (11)$$

which is to label the target state $|g_1\rangle|g_2\rangle$. Similarly, target state $|e_1\rangle|e_2\rangle$ can be labeled by setting $b = \frac{\pi}{2}$, $h = 4m + 3$ (m is an interger), i.e., $t_2 = \frac{\pi}{4\lambda}$, $\frac{\Omega}{\lambda} = 4m + 3$, which yields

$$H^{\otimes 2}U_I(t_2)H^{\otimes 2} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I_{e_1e_2} \quad (12)$$

As for the target state $|g_1\rangle|e_2\rangle$ or $|e_1\rangle|g_2\rangle$, they can be achieved by slight modification of the above operations as follows[13],

$$\sigma_{x,2}I_{g_1g_2}\sigma_{x,2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I_{g_1e_2} \quad (13)$$

$$\sigma_{x,2} I_{e_1 e_2} \sigma_{x,2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I_{e_1 g_2} \quad (14)$$

where $\sigma_{x,2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is the NOT gate acting on atom 2. Therefore, Given a specific λ , by choosing an appropriate Ω , we can generate all the two-qubit operations necessary in the two-qubit Grover's algorithm.

To carry out our scheme, we consider a cavity, i.e., a Fabry-Perot resonator with a single mode and a standing-wave pattern along the cavity axis [5], as shown in Fig. 1. Two atoms, first simultaneously prepared in box B into high-lying circular Rydberg state denoted by $|g_1\rangle|g_2\rangle$, after $H^{\otimes 2}$ operation, are in the initial average state. Then they undergo the operations in Fig. 1 from the left to the right. For searching $|g_1\rangle|g_2\rangle$ or $|e_1\rangle|e_2\rangle$, our implementation is straightforward because the atoms interact with the cavity and the classical field simultaneously. While to search $|e_1\rangle|g_2\rangle$ or $|g_1\rangle|e_2\rangle$, since NOT gates are only performed on atom 2, we have to employ an inhomogeneous field to distinguish the two atoms. This can be done by the same trick as in Ref. [5], i.e., introducing an inhomogeneous electric field in the two regions respectively where $\sigma_{x,2}$ takes action. Finally the atoms are separately read out by the state-selective field-ionization detectors D_1 and D_2 .

Now we briefly discuss the experimental possibility of our proposal. Considering two Rydberg atoms with principal quantum numbers 50 and 51 and the radiative time $T_r = 3 \times 10^{-2}$ s, we assume that the atom-cavity coupling strength g is $25 \times 2\pi$ kHz [7, 14], $\delta = 20 \times g$, and $\Omega \approx 20 \times \delta$. Direct calculation shows that the time for the two U_I operations is 4.0×10^{-4} s. As the time for single-qubit operations is negligible, the implementation time in the cavity in Fig. 1 is much shorter than the radiative time T_r . Moreover, the atomic state evolution is independent of the cavity field state. So our proposal is realizable with presently available cavity QED techniques. Furthermore, it should be pointed out that the Rabi frequency Ω during the two-qubit gate is about $10 \times 2\pi$ MHz and should be slightly adjusted to satisfy the condition $\frac{\Omega}{\lambda} = 4m + 1$ or $\frac{\Omega}{\lambda} = 4m + 3$ mentioned above.

To check the validity of our scheme more strictly, we numerically simulate the time evolution of the system for finding the target state $|g_1\rangle|g_2\rangle$. As shown in Fig. 2 (a), if

we assume that the cavity is initially in a Fock state $|n\rangle$, the probability of finding the target state $|g_1\rangle|g_2\rangle$ slightly decreases with the increase of the photon number. Even for $n = 10$, however, the fidelity can still be 99.2%, which means the whole process is almost independent of the cavity state. Fig. 2 (b) presents the influence of the imperfect operations on the fidelity. For simplicity, we assume the initial cavity state to be $|5\rangle$ and the same imperfection in each pulse. We see from the plot that even for 7% pulse error, the fidelity is still larger than 90%.

In summary, we have proposed a simple scheme for implementing two-qubit Grover search algorithm in cavity QED. By adding a strong resonant classical field during the two-qubit operation, we cancelled the phonon-number dependent Stark shift. Thus our scheme is immune to both cavity decay and the thermal field. In addition, different from Ref. [5], in our proposal, two levels $|g\rangle$ and $|e\rangle$ of each atoms are employed to encode the qubits (i.e., qubit definitions are the same for both atoms) and all the operations except the two NOT gates acting on atom 2 are imposed on the two atoms simultaneously, which may make the experimental implementation easier.

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- [12] To reach Eq. (9), the matrix elements in Eq. (8) should meet following condition: $\frac{1}{2} + \frac{1}{2} \cos(bh)e^{-ib} = \frac{i}{2} \sin(bh)e^{-ib} = \frac{1}{2} - \frac{1}{2} \cos(bh)e^{-ib}$. It's easy to have $bh = \frac{\pi}{2} + N\pi$ ($N = 0, 1, 2, \dots$). Therefore, if $N = 2m$ with m being an integer, and $b = \frac{\pi}{2}$, i.e., $\lambda t = \frac{\pi}{4}$, we have $\Omega t = \frac{\pi}{4} + m\pi$, $h = \frac{\Omega}{\lambda} = 4m + 1$; If $N = 2m + 1$ and $b = \frac{3\pi}{2}$, i.e., $\lambda t = \frac{3\pi}{4}$, we obtain $\Omega t = \frac{\pi}{4} + m\pi + \frac{\pi}{2}$, $h = \frac{\Omega}{\lambda} = \frac{4m+3}{2}$. Choosing the first solution, we get Eq. (9) in the shortest time $t_D = \frac{\pi}{4\lambda}$.
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Figure caption

FIG. 1. Schematic plot of our proposal, where the two atoms are initially prepared in the average state, and go through the cavity from the left to the right with the same velocity. The three operations $H^{\otimes 2}$, $U_I(t)$, $H^{\otimes 2}$ constitute the conditional phase gate $I_{g_1g_2}$ or $I_{e_1e_2}$, and the two NOT gates acting on atom 2, i.e., $\sigma_{x,2}$, added on the two ends of above three operations are for generating $I_{g_1e_2}$ or $I_{e_1g_2}$. The next operation $U_I(t)$ is for the diffusion transform D . D_1 and D_2 are detectors to check the states of atoms 1 and 2 respectively.

FIG. 2. Numerical results for the fidelity of our scheme, in which $g = 25 \times 2\pi$ kHz, $\delta = 20 \times g$, $\Omega = 16001 \times \lambda$ and the target state is $|g_1\rangle|g_2\rangle$. (a) Fidelity for different initial cavity Fock state $|n\rangle$. (b) Given the initial cavity state $|5\rangle$, the fidelity with respect to the pulse imperfections.



